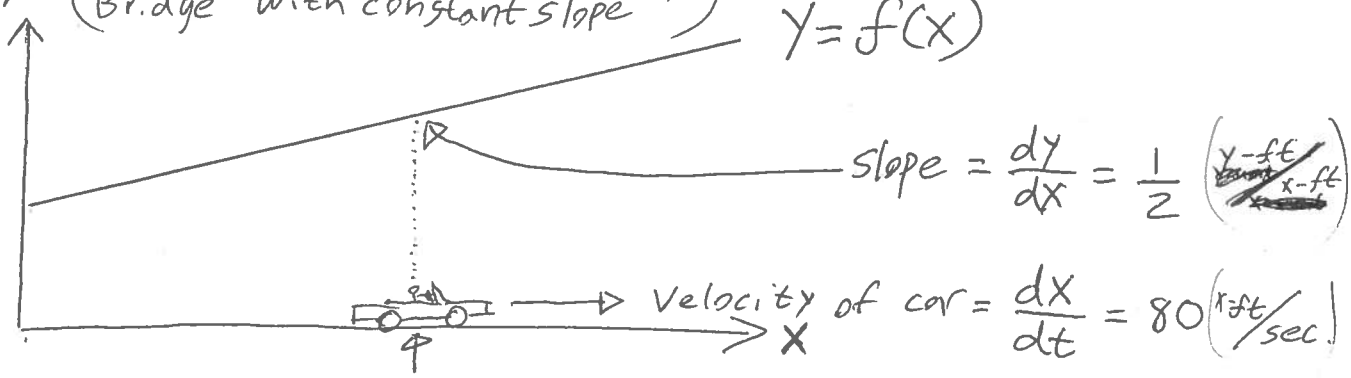
Chain Rule

(1) (Car with constant Velocity
Bridge with constant slope)



(a.) How fast is the bridge rising above car in y ft/sec.? $\left(\frac{dy}{dt} = ?\right)$

$$\frac{dy}{dt} = \underbrace{\frac{dy}{dx}}_{\text{slope of Bridge}} \cdot \underbrace{\frac{dx}{dt}}_{\text{Veloc. of car}} = \left(\frac{1}{2}\right)(80) = \boxed{40\left(\frac{y\text{-ft}}{\text{sec}}\right)}$$

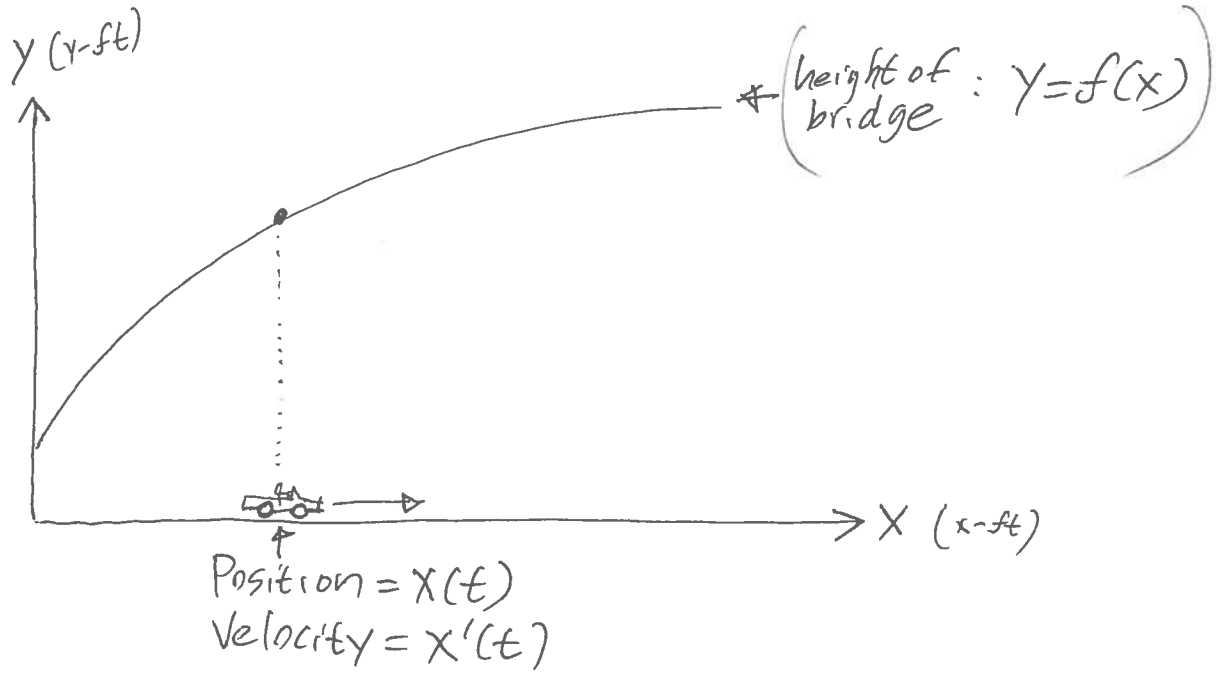
(b.) What if car's speed was not constant?

Velocity at time $t = x'(t) = \frac{dx}{dt}$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \boxed{\frac{1}{2} \cdot x'(t) \left(\frac{y\text{-ft}}{\text{sec}}\right)} \leftarrow \left(\begin{array}{l}\text{different}\\ \text{rates at}\\ \text{different}\\ \text{times}\end{array}\right)$$

(2) See next Page

(2)



Q: How fast is bridge rising above car (in y-ft/sec) at a given time?

A: Depends on car's velocity at time t and on slope of bridge above car's location.

Bridge rise rate = (Slope of bridge above car's location)(Velo. of car)

Height of Bridge (at time t) above car = $f(x(t))$

Velocity of car at time t = $x'(t)$

So, slope of Bridge above car at time t = $f'(x(t))$

Bridge Rise rate = $f'(x(t)) \cdot x'(t)$

$[f(x(t))]' =$

$$y\text{-ft/sec} = \frac{y\text{-ft}}{x\text{-ft.}} \cdot \frac{x\text{-ft}}{\text{sec}}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$